

Probabilistic Graphical Models

Lectures 15

Map Inference - Max Marginals

Inference

- Belief propagation (Sum-product)
- MAP inference



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MAP inference



MAP Inference

$$P(A, B, C, D, E)$$

$$X^* = \operatorname{argmax}_X P(X)$$

$$X^* = \operatorname{argmax}_X P(X | Y=y)$$

$$A^*, B^*, C^*, D^*, E^* = \operatorname{argmax}_{A, B, C, D, E} P(A, B, C, D, E)$$

find the joint assignment to

$$X_1, X_2, \dots, X_n$$

that maximizes $P(X_1, X_2, \dots, X_n)$.



Max Marginal



for a function $f(A, B)$ [may or may not be
a probability distribution]

$$g(A) = \max_B f(A, B)$$

Max Marginal



$$g(A) = \max_B f(A, B)$$

$$A \in \{0, 1\} \quad B = \{1, 2, 3\}$$

Example

	B=1	B=2	B=3	
A=0	0.3	0.4	0.3	MAX \Rightarrow
A=1	0.8	0.2	0.3	

\Downarrow MAX

	B=1	B=2	B=3	
	0.8	0.4	0.3	MAX \Rightarrow 0.8

$$h(B) = \max_A f(A, B)$$

Example

Max Marginal



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Example

$$f(A, B) = A B$$

$$A, B \in \{0, 1\}$$

$$g(A) = \max_B f(A, B) = \max_B A B = \begin{cases} 0 & A=0 \\ 1 & A=1 \end{cases}$$

$$\Rightarrow g(A) = A$$

Max Marginal



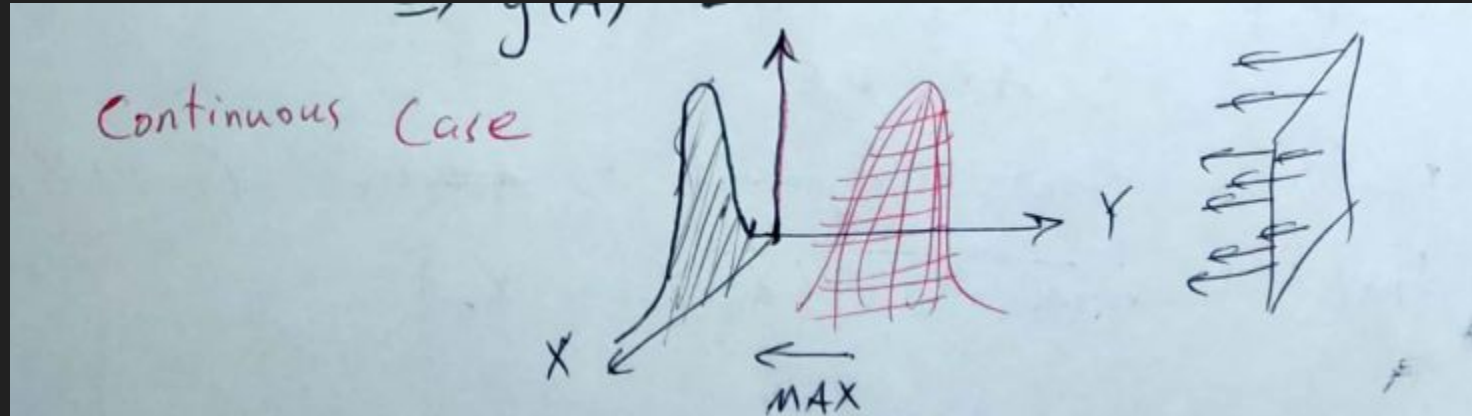
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$$\begin{aligned} \circ f(A, B) &= 1(A \neq B) & A, B &= \{0, 1\} \\ g(A) &= \max_B 1(A \neq B) = \begin{cases} 1 & A=0 \\ 1 & A=1 \end{cases} & \begin{array}{|c|c|} \hline 0 & 1 \\ \hline 1 & 0 \\ \hline \end{array} & \begin{array}{|c|} \hline 1 \\ \hline 1 \\ \hline \end{array} \\ \Rightarrow g(A) &= 1 & \text{MAX} \Rightarrow \end{aligned}$$

Max Marginal



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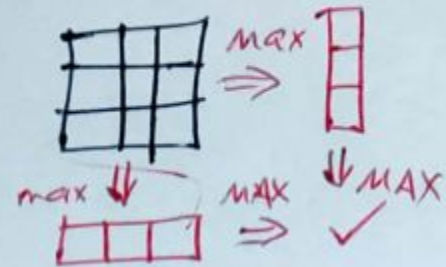


Max Marginal



$$\max_B \max_A f(A, B) = \max_A \max_B f(A, B)$$

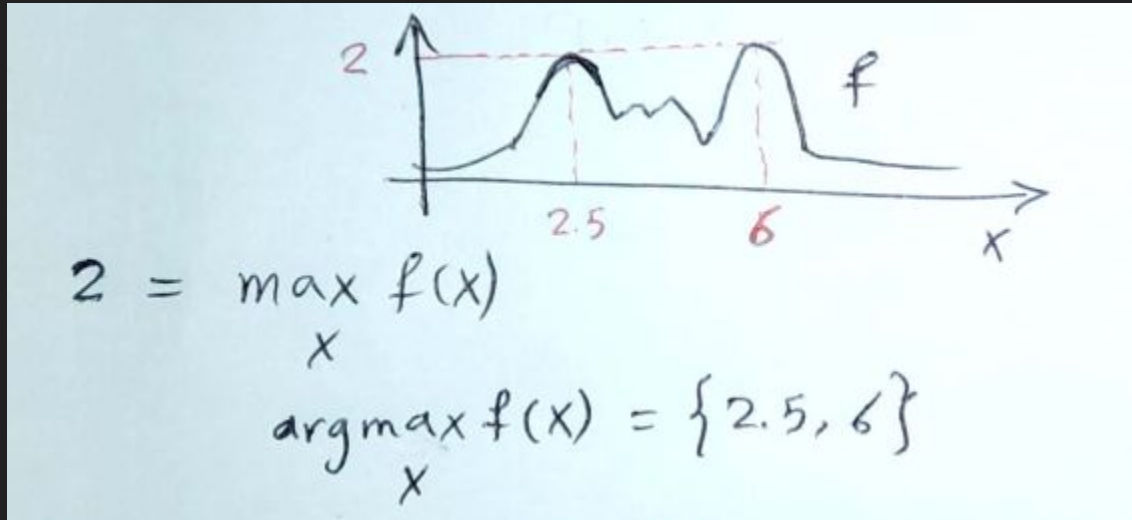
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Max Marginal



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Max Marginal



$$A^*, B^*, C^*, D^*, E^* = \underset{A-E}{\operatorname{argmax}} f(A, B, C, D, E)$$

Argmax might not be unique.

$$f(A, B) = 1 (A \neq B)$$

	A=0	A=1
B=0	0	1
B=1	1	0

$$\underset{A, B}{\operatorname{argmax}} f(A, B) \begin{cases} \rightarrow A=0, B=1 \\ \rightarrow A=1, B=0 \end{cases}$$

Max Marginal



$$\underline{A^*}, B^*, C^*, D^*, E^* = \underset{A, B, C, D, E}{\operatorname{argmax}} P(A, B, C, D, E)$$

Assume that (A^*, B^*) is unique.
the solution

1-obtain max-marginal $g(A) = \max_{B, C, D, E} P(A, B, C, D, E)$

2-compute argmax $\underline{A^{**}} = \underset{A}{\operatorname{argmax}} g(A)$ (also unique) ^{نیض}

$$A^* \stackrel{?}{=} A^{**} \quad \text{YES}$$